Multifractal analysis of time series: concepts, methodology, and practical issues

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The workshop will start with a general introduction to fractal analysis, in which the basic concepts like the Hurst exponent, fractal dimension, singularity spectrum, etc. are defined together with their practical application to model and real-world data. Then we will focus on time series and learn how to identify their fractal and multifractal properties by means of the most popular dedicated tools, among which the methods based on detrended fluctuations, detrended moving-average, wavelet transform, and cumulative mass are the most important.

We will focus not only on the methodology itself, but will also learn about the most interesting results reported in literature that were obtained with multifractal analysis. Obviously, the knowledge of how some methodology works is by no means complete without a deeper insight into its weak points and limitations - they will also be discussed. Next we will look into details, gain an understanding of certain subtle but meaningful effects related to data structure, like the singularity spectrum asymmetry, the distortions caused by nonstationarities, and the false multifractality related to non-Gaussian fluctuations.

Finally, we will focus on data modelling and review the most important stochastic processes that can reproduce both multiscaling and some other properties of empirical data.

At the end of my workshop, students will understand the concept of multifractality and its importance for a proper description of natural complex systems, they will be ready to carry out an advanced multifractal analysis of empirical time series, distinguish between the genuine multiscaling related to long-range auto- and cross-correlations and spurious effects that resemble multiscaling, and will be familiar with multifractal stochastic processes that are of significance in modeling of complex systems.

References

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